Compressive Sensing for Multi Sensor Systems

Matthias Weiß

Fraunhofer Institut für Hochfrequenzphysik und Radartechnik

FHR Passive Sensoren und Klassifizierung

Fraunhoferstraße 20, 53343 Wachtberg, Germany

E-Mail: matthias.weiss@fhr.fraunhofer.de

Abstract

The aim of this lecture is to provide a short introduction to compressive sensing (CS) techniques and its application to radar systems and networks. In spite of the fact that CS theory is a very young mathematical framework for solving sparsely populated linear systems (2004, [3]), it currently represents a revolution in signal processing and sensor systems. The reason for this can be seen in the potential of CS techniques in reducing the number of required samples and/or of the number of sensors without degrading the performance of the system. A few properties of CS in the area of radar and fusing data in sensor networks will be discussed and several examples will be given throughout this paper to prove the presented concept.

1.0 INTRODUCTION

Surveillance and reconnaissance systems using radar sensors offer several advantages compared to optical systems. For instance radar sensors operate independently of day light, and they are not influenced by clouds, fog, and dust. Also electromagnetic waves can penetrate non metallic material and can therefore be used to detect objects concealed in a forest [1]. Another big advantage of radar is that the range resolution does not decrease with the distance between sensor and the scene and due to the coherent signal processing more information can be extracted from the received echo.

---

[0]Weiß, M. (2013) Compressive Sensing for Multi Sensor Systems. In *Radar and SAR Systems for airborne and space-based surveillance and reconnaissance* (pp. 1-1 — 1-23). STO-LS-SET-191, Paper 1. Neuilly-sur-Seine, France: STO. Available from: http://www.cso.nato.int/abstracts.aps.

Many state of the art radar systems operate with large frequency bandwidths and use phased array techniques to achieve a highly flexible and adaptable surveillance system. Due to the large instantaneous bandwidth and huge number of receive elements a lot of data is generated and has to be processed. However typical radar scenes consist only of a small number of existing targets. Not withstanding this, the traditional methods need to process all data to estimate range and Doppler for a few targets.

Surveillance systems where transmit and receive nodes are distributed over an area, sometimes called multiple-input and multiple-output (MIMO) sensor systems, exhibit several advantages compared to single sensor systems. For instance, due to the joint signal processing of a radar network a higher spatial resolution can be achieved. Likewise target detection and Doppler estimation are improved and the handling of multiple targets is enhanced. This only became possible in recent years by the technological improvement of high-speed links, which are essential for transferring data between the nodes and the central processing stage via cable and/or wireless connections. In addition to that digital modulation techniques have opened the realization of distributed radar networks. There is no further need for a surveillance channel as an ideal reference signal can be created from the distorted received signal, which consists of a mix of direct signal, multi-path signals, and the echo in the digital domain. For passive and multistatic radar systems special tracking techniques have been developed to distinguish between targets and ghost objects [2]. These techniques rely on a central signal processing scheme for detecting targets and estimating their parameters and therefore high-speed links are essential to establish these techniques.

Many attempts have been made to reduce the required data rate for these systems. With the beginning of the 21th century the new sensing/sampling paradigm, called compressive sensing (CS), has been developed which overcomes the Nyquist-Shannon sampling theorem and helps in the area of wide band systems and sensor fusion. The CS theory claims that it can recover specific signals from far fewer samples than required by the traditional methods. To achieve this CS relies on two assumptions: the reconstructed signal is sparse in some orthonormal basis (e.g. wavelet, Fourier) or tight frame (e.g. curvelet, Gabor) and the columns of the sensing matrix are uncorrelated [3].

The focus of this paper is to give a short overview of compressive sensing applied to high resolution radar and to data fusion in distributed radar networks.

2.0 NOTATIONS AND DEFINIITONS

Throughout this paper boldface variables represent vectors and matrices while non-boldface variables represent functions with a continuous domain. For a natural number N the set $[N]$ is defined by $[N] := \{1, \ldots, N\}$. The cardinality of a set $\boldsymbol{A}$ is denoted by $card(\boldsymbol{A})$ and is a measure of the number of elements of $\boldsymbol{A}$. For a real number $p \geq 1$ the ℓ_p-norm or p-norm of a vector $\boldsymbol{x} \in \mathbb{C}^N$ is

defined by:

$$\|x\|_p \;\; = \;\; \left(\sum_{n=1}^{N} |x_n|^p\right)^{1/p} \quad , \quad 0 < p < \infty \tag{1}$$

$$\tag{2}$$

The ℓ_∞ or *maximum norm* is the limit of the ℓ_p norms for $p \to \infty$ and is describes the greatest distance between two vectors along any coordinate dimension and is equal to:

$$\|x\|_\infty \;\; = \;\; \max\{|x_1|, \ldots, |x_n|\} \tag{3}$$

The Euclidean norm, or the length of vector $\boldsymbol{x}$, is according to the above definition (1) the ℓ_2-norm and the ℓ_1-norm is the norm which corresponds to the so called *Manhattan distance*.

For $p \to 0$ we use the ℓ_0-"norm" definition from [4] as a count of the non-zero elements in $\boldsymbol{x}$ and is equal to $card(n \in [N] : x_n \neq 0)$.

If $\boldsymbol{x_1}, \ldots, \boldsymbol{x_N}$ are columns vectors then $vec(\boldsymbol{x_1}, \ldots, \boldsymbol{x_N}) := (\boldsymbol{x_1}^T, \ldots, \boldsymbol{x_N}^T)^T$ denotes a column vector built by stacking all $\boldsymbol{x_n}$, $n \in [N]$. The n-th element of vector $\boldsymbol{x}$ is denoted by x_n.

For a given matrix $\boldsymbol{A}$, $\boldsymbol{A}^T$, $\boldsymbol{A}^H$, and $Tr(\boldsymbol{A})$ denote the transpose, conjugate transpose, and trace. The element in the i-th row and j-th column of $\boldsymbol{A}$ is denoted by a_{ij}, and $\boldsymbol{a}_n$ stands for the n-th column of $\boldsymbol{A}$ and $\boldsymbol{a}^m$ for the m-th row, respectively.

The Hadamard product, also known as Schur product or element-wise product, of two matrices $\boldsymbol{C} = \boldsymbol{A} \circ \boldsymbol{B}$ with identical dimensions yield a matrix with the same dimension where each element c_{ij} is a product of the elements a_{ij} and b_{ij} of the original matrices.

3.0   BASIC IDEA

Compressive sensing is a recently developed mathematical framework [3]–[7] with the primary purpose of reconstructing the sparse signal $\boldsymbol{s}$ from a linear measurement with noise $\boldsymbol{y} = \boldsymbol{As} + \boldsymbol{n}$, as it will be always the case for real applications. The vector $\boldsymbol{s} \in \mathbb{C}^{N \times 1}$ describes the sparsely populated scene and the measurements obtained by a linear sensor are collected in the M-dimensional vector $\boldsymbol{y}$. The sensing matrix $\boldsymbol{A}$ is a $M \times N$ dimensional matrix and defines how each element from the scene s_i contributes to the measurement $\boldsymbol{y}$.

CS expects that $M < N$, which prevents that $\boldsymbol{s}$ can be reconstructed by simply inverting $\boldsymbol{y} = \boldsymbol{As}$ as this leads to a underdetermined system of linear equations. If the sparse signal $\boldsymbol{s}$ of dimension N has K-sparse representation ($\|s\|_0 = K \ll N$) and is compressible, which means that the vector coefficients are composed of a few large coefficients and other coefficients with small value ($\|\boldsymbol{s} - \boldsymbol{s}(K)\|$ decreases quickly to zero with growing K), CS is capable of recovering the sparse signal

s exactly with a very high probability from fewmeasurements by solving a convex ℓ_1 optimization problem of the form:

$$\min \|s\|_1, \quad \text{subject to} \quad \|y - As\|_2 < \sigma \quad . \tag{4}$$

Well known are the Basis Pursuit Denoising (BPD) [8], the Orthogonal Matching Pursuit (OMP) [9], the Compressive Sampling Matched Pursuit (CoSaMP) [10], and the SPGL1 [11] algorithms to solve the above equation.

Owing to these attractive properties CS found a lot of attraction in the field of radar with its sparse scenes over the past years. One of the earliest papers on CS applied to radar is from Baraniuk [12]. Nowadays there are numerous research projects going on to further investigate compressive sensing applied to high resolution radar, interferometric SAR, ISAR, Moving Target Indication (MTI), and DOA estimation, for instance.

It has been shown that CS provides a guaranteed stable solution of the reconstructed sparse signal s for a sensing matrix A if it satisfies the following three properties: null space property, restricted isometry property, and the matrix column coherence.

4.0   PROPERTIES OF THE SENSINGS MATRIX

To solve the underdetermined equation system with CS techniques two questions have to be solved. The first one deals with the sensing matrix A and its design to preserve the information of the measurements in a sparse signal s. The other one is how the sparse signal s can be recovered from the measurements y. If signal s is sparse or compressible, a sensing matrix A with dimension $M \ll N$ can be designed in such a way that CS reconstruction algorithms can recover the original signal s accurately and efficiently.

To ensure that CS reconstruction algorithms lead to a perfect solution we have to take care designing the sensing matrix A. The following subsections consider a number of desirable properties that the sensing matrix A should have.

4.1   Null space property (NSP)

For an exactly sparse vector s the spark-function of the sensing matrix A, which delivers the smallest number of linearly dependent columns in A, provides a complete characterization if the recovery is possible [13].

$$\mathrm{spark}(A) = \min_{s \neq 0} \|s\|_0 \quad \text{subjet to} \quad As = 0 \tag{5}$$

But for approximately sparse signals a more restrictive conditions has to be introduced on the null space of A [14], [15]. It must be ensured that the null space $\mathcal{N}(A) = s : As = 0$ does not contain column vectors that are too compressible moreover to sparse column vectors.

Consider that s_0 is the solution of $y = As$, then any solution s' of this equation can be written as $s' = s_0 + v$ with $v \in \ker A$ and

$$\mathcal{N}(A) = \text{Null}(A) = \ker(A) = \left\{ v \in \mathbb{C}^N, Av = 0 \right\} \quad . \tag{6}$$

If $\Delta : \mathbb{R}^M \to \mathbb{R}^N$ represent the selected recovery method the matrix A satisfies the null space property (NSP) of order K if there exists a constant $C > 0$ such that [16]:

$$\|\Delta(As) - s\|_2 \leq C \frac{\min \|s - \hat{s}\|_1}{\sqrt{K}} \tag{7}$$

for all s, with $\hat{s}$ the approximation of the signal s.

The NSP describes that the column vectors in the null space of A should not be too concentrated on a small subset of indices. If the sensing matrix A fulfils the NSP it guarantees exact recovery of all possible K-sparse signals and it ensures a degree of robustness to non-sparse signals that directly depends on how well the signals are approximated by K-sparse vectors.

As it is difficult to determine the NSP the restricted isometry property, as introduced by [17], has become a more popular tool in compressive sensing theory.

4.2 Restricted isometry property

The NSP is necessary and sufficient for establishing guarantees of Eqn. (7), but only for the noise-free case. When measurements contain noise or are corrupted by some error as caused, for example by quantization or non-linear devices, a stronger constraint has to be chosen. Candès and Tao introduced in [17] the following isometry condition on matrix A and established its important role in CS.

An $M \times N$ matrix A satisfies the restricted isometry property (RIP) of order K if there exists a $\delta_K \in (0, 1)$ such that [18]-[22]:

$$(1 - \delta_K)\|s\|_2^2 \leq \|As\|_2^2 \leq (1 + \delta_K)\|s\|_2^2 \tag{8}$$

for all K-sparse vectors $s \in \mathbb{C}^N$. When δ_K is less than 1 this RIP imply that all of the submatrices of A with K-columns are well-conditioned and close to an isometry. If $\delta_K \ll 1$ then there is a large probability to reconstruct the K sparse signal s with the sensing matrix A.

4.3 Coherence

The spark, NSP, and RIP criterion all provides a guarantee for the recovery of a K-sparse signal, but any of these properties are hard to verify for a general matrix A as $\binom{n}{k}$ submatrices has to be considered during the computation. In many cases it is preferred to determine a characteristic of A that is much easier to compute and yields more practical recovery guarantees. Such a property is the coherence or mutual coherence of matrix A [13]-[15]. It is defined as the maximum absolute value of

the cross-correlations between the N columns of matrix $\boldsymbol{A} \in \mathbb{C}^{M \times N}$. This matrix coherence should not be confused with the coherence of the sensor system. Formally, let $\boldsymbol{a}_1, \ldots, \boldsymbol{a}_N$ be the columns of the matrix $\boldsymbol{A}$. The coherence of $\boldsymbol{A}$ is then defined as:

$$\mu(\boldsymbol{A}) \quad = \quad \max_{1 \leq i \neq j \leq N} \frac{|< \boldsymbol{a}_i^*, \boldsymbol{a}_j >|}{\|\boldsymbol{a}_i\|_2 \, \|\boldsymbol{a}_j\|_2} \tag{9}$$

and $|<,>|$ is the product between any two columns $\boldsymbol{a}_i, \boldsymbol{a}_j$ with $1 \leq i \neq j \leq N$. It can be shown that the coherence $\mu(\boldsymbol{A})$ is always in the range of $\mu(\boldsymbol{A}) \in \left[\sqrt{\frac{N-M}{M(N-1)}}, 1 \right]$. The lower bound is also known as the Welch bound and is for $N \ll M$ approximately $\mu(\boldsymbol{A}) = 1/\sqrt{M}$ [23]-[24].

In the case of $\mu(\boldsymbol{A}) = 1$ there are at least two columns aligned. This represents the worst case scenario: maximum coherence. The other extreme, when $\mu(\boldsymbol{A}) = \sqrt{(M-N)/N(M-1)}$ the best scenario exists: maximal incoherence.

For a good convergence of the recovery algorithms the coherence μ of the columns of the sensing matrix $\mu(\boldsymbol{A})$ should be < 1.

5.0   RADAR APPLICATIONS

The remaining sections will discuss several compressive sensing applications in the field of radar and in the area of data fusion for distributed sensor networks.

For high-resolution radar CS is usable for pulse compression in the time or frequency domain. It will be shown that this new technique allows reducing the number of data without decreasing the performance of the radar. Another interesting application can be found in the reconstruction of corrupted signals. The performance of CS in the area of spatial sparsity, like antenna arrays for locating signal sources by directional-of-arrival estimation, will also be shown.

One further application discussed in this paper is to use compressive sensing to fuse information from a distributed radar network, which is also called multiple-input multiple-output (MIMO) radar system. If the scene observed by the sensors can be described by a simple linear target state vector then this also works with very diverse sensors as input nodes for CS. It's always a question how to setup the sensing matrix $\boldsymbol{A}$ so all information from the sensors can be described by the single state vector. Constructing the sensing matrix $\boldsymbol{A}$, one has always to keep in mind that it has to fulfill the three mentioned properties NRP, RIP, and coherency to guarantee the reconstruction of the sparse vector $\boldsymbol{s}$ from the measurement $\boldsymbol{y}$

5.1   Pulse compression

The traditional way of how radar works is that it emits a frequency modulated pulse and the reflected signal is received, down-converted, digitized before further signal processing algorithms extract the

information from the almost continuous data stream. For high-resolution chirp radars and their wide frequency bandwidth the pulse compression can be implemented in an analog way, which gets along with a low-rate analog-to-digital converter (ADC), or digital by sampling the whole signal bandwidth with high-speed ADC followed by a fast data storage unit and realizing the matched filter in the digital domain. Due to this, the design of high resolution radar systems is limited by the required high-speed components which are in many cases beyond the state-of-art of what is currently technologically possible or the technique is too expensive.

On the receiver side, compressive sensing can help to overcome data-rate problems as pulse compression is performed by using just a few measurement samples, which avoids the need to continuously sample the received signal and store it. In contrast to the matched filter approach, CS reconstructs the compressed signal from only a few measurements by solving an inverse problem either through a linear program or a greedy pursuit [12]. This changes the radar design dramatically as the demanded ADC bandwidth is reduced and the traditional matched filter processing is replaced by CS as the data rate from the sparse scene with some targets is lot less than the Nyquist-Shannon rate.

5.1.1 Time domain

A radar illuminates the surveillance area with the signal $x_t(t)$. The received signal y is a sum of the reflected signal from target $m = 1, \ldots, M$ with a radar cross section α_m at a distance r_m, which corresponding to a time-delay of $\tau_m = 2r_m/c$. The receiver samples y at $t = t(0), \ldots, t(L)$. For the time $t(l)$ this is:

$$y_l = \sum_{m=1}^{M} \alpha_m \, x_t(t_l - \tau_m) \tag{10}$$

$$\tag{11}$$

The measurement vector y can therefore described by:

$$\boldsymbol{y} = [y_1, \ldots, y_L]^T = \boldsymbol{As} \quad . \tag{12}$$

The target state vector contains all possible target RCS $\boldsymbol{s} = [\alpha_1, \ldots, \alpha_M]^T \in \mathbb{C}^{M \times 1}$ and the sensing matrix $\boldsymbol{A}$:

$$\boldsymbol{A} = \text{vec}(\boldsymbol{x}(\tau_0), \ldots, \boldsymbol{x}(\tau_M)) \tag{13}$$

with $\boldsymbol{x}(\tau_m) = [x_t(t_0 - \tau_m), \ldots, x_t(t_L - \tau_m)]^T$ a time-delayed version of the transmit signal.

Fig. 1 shows the result from a radar transmitting a frequency modulated pulse with a bandwidth of 147 MHz. The sample rate is twice the bandwidth. The top diagram shows the given range profile and the second diagram the result from the matched filter with about 3,000 sampled data. The same range profile can be obtained by CS, as depicted in the third top diagram. However, CS is still able to recover the range profile even when the Nyquist-Shannon criteria is not fulfilled by taking only every 60th sample from the measurement, as depicted in the bottom diagram. In contrast the matched filter is not able to determine the right range profile.
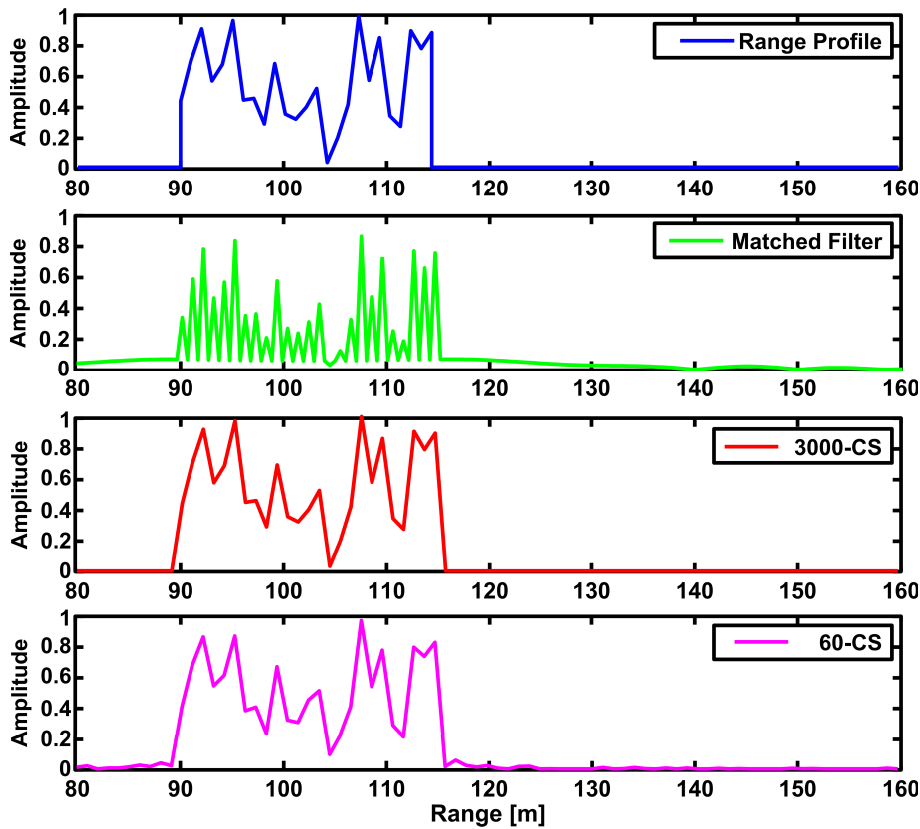
Fig. 1: Pulse compression in the time domain with the matched filter approach and compressive sensing.

5.1.2 Frequency domain

Let a radar illuminate a scene with M discrete $f_1, \ldots, f_m$ frequencies. The received signal can then be described by:

$$y_m = \sum_{n=1}^{N} \alpha_n \, e^{-j2\pi f_m \tau_n} \tag{14}$$

$$\boldsymbol{y} = [y_1, \ldots, y_M]^T = \boldsymbol{As} \tag{15}$$

with a target state vector $\boldsymbol{s} = [\alpha_1, \ldots, \alpha_M]' \in \mathbb{C}^{M \times 1}$ and a sensing matrix $\boldsymbol{A}$ of

$$\boldsymbol{A} = \begin{pmatrix} e^{j2\pi f_1 \tau_1} & \cdots & e^{j2\pi f_1 \tau_n} \\ \vdots & \ddots & \vdots \\ e^{j2\pi f_M \tau_1} & \cdots & e^{j2\pi f_M \tau_n} \end{pmatrix} \tag{16}$$

Fig. 2 shows the result from such a discrete radar system with 1024 frequency points uniformly distributed over a bandwidth of 1.5 GHz. The top diagram shows the result when all frequencies

are used for calculating the inverse Fourier transformation, which represents the given range profile very well. The same result can be obtained by using compressive sensing techniques, as depicted in the second diagram. In the case of only 100 random frequencies selected from the measurements this changes dramatically, as depicted in the two lower diagrams. In contrast to the FFT with 100 frequencies CS is still able to estimate the range profile with a high accuracy, as depicted in the bottom diagram.
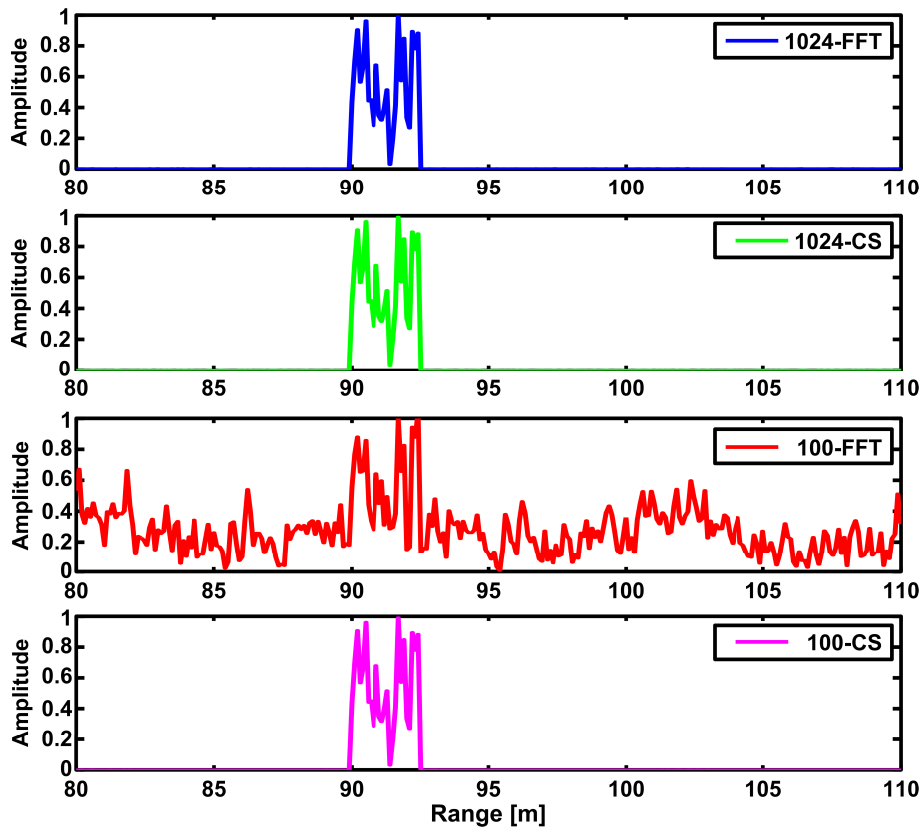
Fig. 2: Comparison between traditional pulse compression (matched filter) and CS for various number of measurement samples.

5.1.3 Reconstruction of corrupted signals

The above examples already show the benefits one can obtain by implementing CS techniques in the receiver. Another promising investigation of CS is the potential of recovering a signal which shows a corrupted frequency band, for instance demanded by a frequency licence agency to avoid any conflicts between the radar unit and other existing electromagnetic receiving units [25].

An example is shown in Fig. 3.
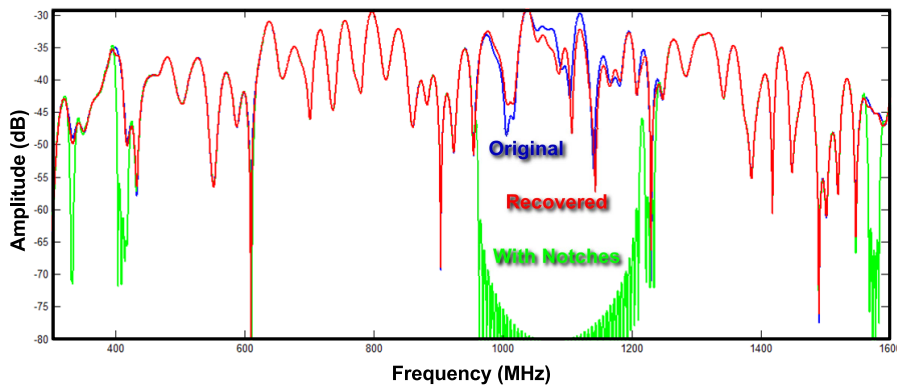
Fig. 3: Recover of a signal with spectral gaps using CS (courtesy by Nguyen, CoSeRa 2012, [25]).

5.2 Direction-of-Arrival estimation

Direction-of-Arrival (DOA) estimation is used to determine the bearing of an impinging signal on a sensor or antenna array. With only two separated locally DOA receivers, signal sources can be accurately located and tracked. Due to its robust technique it is widely used in both civilian and military applications. There exists a variety of classical methods to estimate DOA, which can be subdivided into nonparametric and parametric spectral analysis methods. In the first case one can find the periodogram, correlogram and Capon. Maximum likelihood estimation (ML), MUSIC (MUltiple SIgnal Classification), and ESPRIT (Estimation of Signal Parameters using Rotational Invariance Techniques) are parametric approaches. If there are multiple targets present ML has its particular problems in estimating the correct number of targets and has then a high computationally load due to multiple calculation of the correlation matrix between each receive signal [26]-[27].

Approaches based on CS for DOA estimation can help to overcome some of the existing limitations. For instance, this increase the ability to resolve closely spaced signal sources, the sensitivity against correlated sources is improved, and fewer samples are needed for an accurate estimation.

Let's assume that DOA estimation is performed with an uniform linear antenna array with elements at positions: $p_1, \ldots, p_M$. Let $k_r = 2\pi/\lambda$ be the wave number and $u \in [-1, 1]$ the directional cosine, as depicted in Fig. 4

If the grid directional cosines is denoted by $u_1, \ldots, u_N$ where the signal sources with a signal strength of s_n are located

$$y_m(t) = \sum_{n=1}^{N} s_n e^{-j k_r u_n p_m} \tag{17}$$

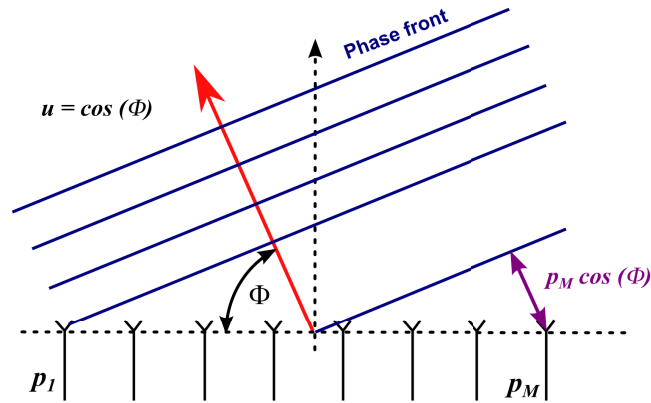$$\boldsymbol{y}(t) = [y_1(t), \ldots, y_M(t)]^T = \boldsymbol{A}\boldsymbol{s}(t) \tag{18}$$

Fig. 4: Linear antenna array

with $\boldsymbol{s}(t) = [s_1, \ldots, s_N]^T$ the signal strength vector of size $(N \times 1)$ and a sensing matrix $\boldsymbol{A}$:

$$\boldsymbol{A} = \begin{pmatrix} e^{-j\,k_r\,u_1\,p_1} & \ldots & e^{-j\,k_r\,u_1\,p_M} \\ \vdots & \ddots & \vdots \\ e^{-j\,k_r\,u_N\,p_1} & \ldots & e^{-j\,k_r\,u_N\,p_M} \end{pmatrix} \tag{19}$$

If the number of targets or signal sources is known and less than the number of antenna elements Eqn. (18) is an overdetermined system of linear equations and can easily be solved by inverting the matrix when the DOA's are known. But the DOA's are unknown and need to be determined. Therefore the directional-of-arriving vector $\boldsymbol{s}$ of size N is sparse and compressive sensing techniques can be applied to solve the underdetermined linear system. To enhance the robustness and accuracy multiple time samples are combined into a single measurement, as proposed in [28].

For verification we consider 5 narrowband far-field sources located at angles 30°, 45°, 50°, 100°, and 130° respectively. The plane waves impinge on an uniformly linear antenna array of $M = 40$ sensors equally spaced at the half wavelength $d = \lambda/2$. Each sensor acquire $L = 200$ samples and the signal-to-noise ratio is 10 dB. As Fig. 5 shows the DOA estimation via CS outperforms conventional
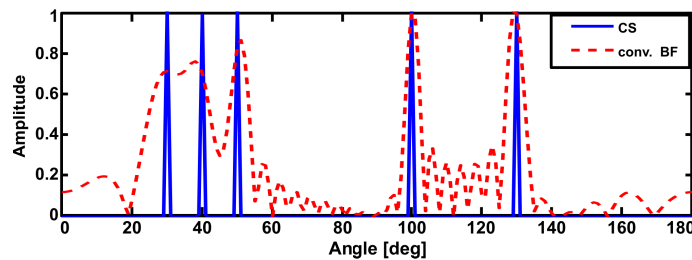

Fig. 5: DOA estimation using a linear antenna array of 30 elements and 5 signal sources. The blue line represents the result obtained by compressive sensing and the red line by conventional beamforming.

beamforming.

5.3   High resolution monostatic Radar

In air surveillance radar the scene is typically sparse populated by only a few targets and is normally characterized by a range/Doppler plane, which helps to distinguish between moving objects and static clutter. If target ζ_k is located at a distance r_k and has a radial velocity which corresponds to a Doppler frequency of $f_D(k)$ the down-converted received signal can be described by:

$$y_k(t) \quad = \quad \alpha_k \, x(t - \tau_k(t)) \, e^{-j2\pi(f_0 - f_D(k))\tau(t)} \, e^{-j2\pi f_D(k)t} \quad , \tag{20}$$

with $\tau(t) = 2r_k(t)/c_0$ the time delay, c_0 the speed of propagation, and f_0 the center frequency of the transmitted signal. The reflection coefficient of the target is denoted by α_k and is a complex value. If the static phase shifts are incorporated into the reflection coefficient and if the target radial speed is $v_r \ll c_0$ the above equation can be rewritten as:

$$y_k(t) \quad = \quad \alpha_k \, x(t - \tau_k) \, e^{-j2\pi f_D(k)t} \quad . \tag{21}$$

In the case that more than one target is present the received signal is the sum over all echoes:

$$y(t) \quad = \quad \sum_k \alpha_k \, x(t - \tau_k) \, e^{-j2\pi f_D(k)t} \quad . \tag{22}$$

The receiver samples and digitizes this signal at time $t = t_0, t_1, \ldots, t_{(L-1)}$. Therefore the measurement vector is $\boldsymbol{y} = [y(t_0), \ldots, y(t_{L-1})]^T$ with L-samples.

To transform Eqn. (22) to $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{s}$ the sensing matrix can easily be constructed by combining time-delay and Doppler matrix in the following way. The time-delay matrix $\boldsymbol{T}$ consists of staggered vectors of the transmit signal with time-delays $\tau_1, \ldots, \tau_M$ ($\boldsymbol{T} \in \mathbb{C}^{L \times M}$) and, hence, is described by:

$$\boldsymbol{T} \quad = \quad \text{vec}(\boldsymbol{x}(\tau_0), \boldsymbol{x}(\tau_1), \ldots, \boldsymbol{x}(\tau_M)) \quad \text{with} \tag{23}$$

$$\boldsymbol{x}(\tau_m) \quad = \quad [x(t_0 - \tau_m), \ldots, x(t_{(L-1)} - \tau_m)]^T$$

The Doppler-shift matrix $\boldsymbol{D} \in \mathbb{C}^{L \times M}$ is the staggered version of the Doppler-vector:

$$\boldsymbol{d}(k) \quad = \quad \left[e^{j2\pi f_D(k)t_0}, \ldots, e^{j2\pi f_D(k)t_{(L-1)}} \right]^T \tag{24}$$

$$\boldsymbol{D} \quad = \quad \text{vec}(\boldsymbol{d}(1), \boldsymbol{d}(2), \ldots, \boldsymbol{d}(M)) \quad , \tag{25}$$

with $f_D(k) \in [-f_{Dmax}, f_{Dmax}]$ the search interval for the Doppler-frequency.

Combining time-delay matrix and Doppler matrix in the following way:

$$\boldsymbol{A} \quad = \quad \text{vec}(\boldsymbol{T} \circ \boldsymbol{D}^0, \boldsymbol{T} \circ \boldsymbol{D}^1, \ldots \boldsymbol{T} \circ \boldsymbol{D}^N) \tag{26}$$

enables us to construct the sensing matrix $\boldsymbol{A}$, which describes the relation between the target state vector $\boldsymbol{s} = [\alpha(\tau_1, f_D(1)), \ldots, \alpha(\tau_M, f_D(N))]^T \in \mathbb{C}^{MN \times 1}$ and the measurement $\boldsymbol{y}$.

$$\boldsymbol{y} \quad = \quad \boldsymbol{A}\boldsymbol{s} \tag{27}$$

With the knowledge that the scene is sparse this underdetermined linear equation system can be solved using CS techniques:

$$\min_{\boldsymbol{s}} ||\boldsymbol{s}||_1 \quad \text{subject to} \quad ||\boldsymbol{A}\,\boldsymbol{s} - \boldsymbol{y}||_2 \leq \sigma \tag{28}$$

5.3.1 Example

In the following simulation a monostatic radar emits a modulated pulse with a pulse length of $\tau_p = 1.5$ ms and a bandwidth of 20 MHz. Therefore the range resolution is $\Delta r = c_0/(2B) = 7.5$ m and the Doppler resolution is $\Delta f_D = 1/\tau_p = 666.6$ Hz. As the center frequency is 2.45 GHz the radial velocity resolution is $\Delta v_r = \lambda \Delta f_D/2 = 40.8$ m/s

For the simulation additive white Gaussian noise (AWGN) was superimposed to the received signal. Fig. 6 shows the result for a signal-to-noise ratio (SNR) of 40 dB. The left diagram shows the
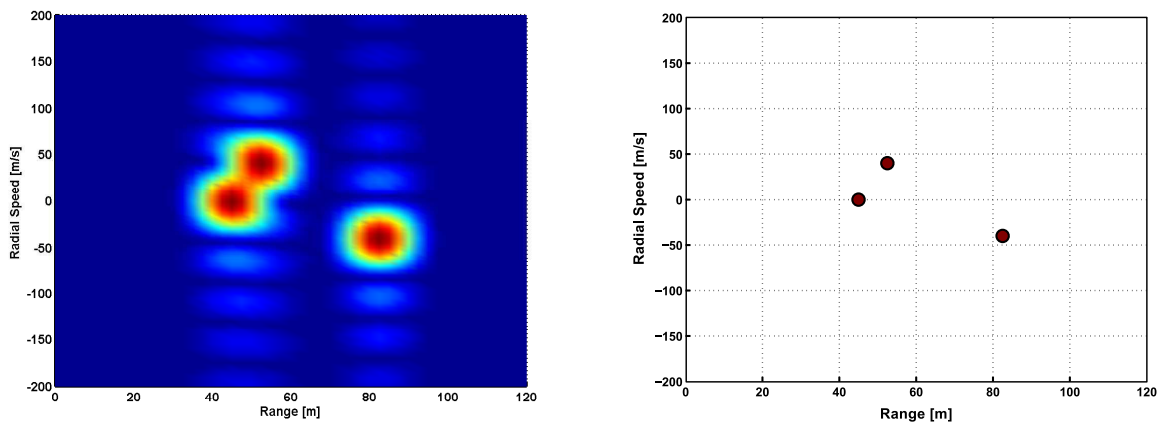
Fig. 6: Simulation result with 3 targets and a signal-noise-ratio of $40\,dB$. Left diagram shows the range/velocity-plane reconstructed by the matched filter approach and right diagram shows the result obtained by CS.

range/velocity-plane one obtains using the traditional matched filter approach and the result from the compressive sensing approach is depicted in the right diagram. Definitely CS is able to determine range and velocity with high accuracy. Reducing the number of samples of the received signal by a factor of 64 does not change this behaviour dramatically for the noise-free case, as shown already in section 5.1.

Fig. 7 illustrates that compressive sensing starts to suffer in the presence of noise. For this example the SNR was $5\,dB$ with a detection level of 0.1. Due to this, several faint false targets appeared, however, the main target can be clearly identified by CS.

5.4 Sensor fusion by CS techniques

In the remaining sections we will focus on the compressive sensing approach of fusing data from a distributed sensor network. This can be in the simplest version a multiple-input and multiple-output (MIMO) system with homogeneous sensors, like radar/sonar systems. In such networks transmit and receive nodes can be co-located or distributed over an area. There exist several publications which
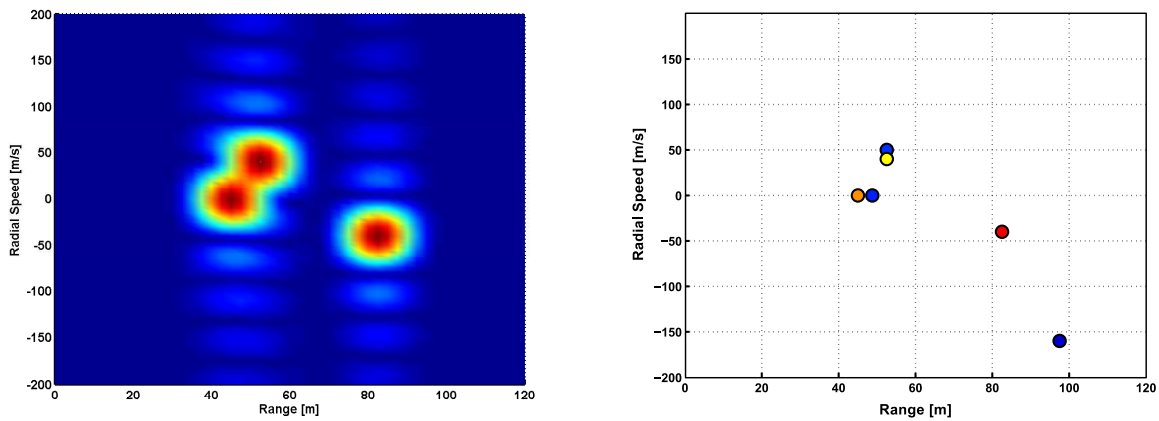
Fig. 7: Same simulation as in the previous Fig. 6 however SNR is now $5\,\mathrm{dB}$. Left diagram shows the range/velocity-plane reconstructed by the matched filter approach and right diagram shows the result obtained by CS.

show that with a distributed network of homogeneous sensors bi- and multistatic constellations are possible. Combining the information from all transmit-receive combinations enhance the detection and tracking performance of the overall system dramatically [29]-[31].

In the following we consider a distributed sensor network, consisting of $M \cdot N$ transmit-receive-pairs, and CS will be applied to estimate the target state vector with high accuracy even with much less measurement data than the Nyquit-Shannon rule demands. Within a multistatic system targets are illuminated from different aspect angles and they show a variable reflection coefficient. Hence, it will be difficult to estimate the target parameters for a coherent system. To overcome this problem we allow that the target parameters, measured by different sensor pairs, are uncorrelated. The only assumption that is made for sensor fusion is that the targets are at the same location, after the sensor data have been transformed into a common x/y-coordinate system [32]. In this common sensor coordinate system targets posses the same target state (position, speed of velocity, direction of movement). However, each sensor will recognize the target with a different amplitude and phase as the radar cross section depends on the aspect angle.
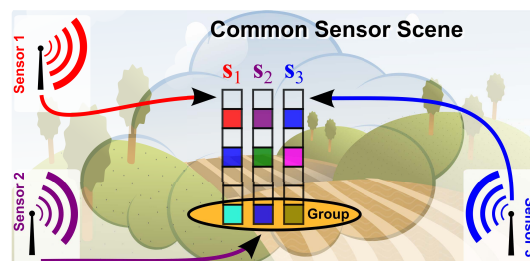

Fig. 8: Principle of diverse sensor systems and sensor fusion using CS group sparsity (block sparsity).

This knowledge leads us to form groups for each target state containing only the corresponding measurements from all contributing sensors, as depicted in Fig. 8. If a target exist all members of the corresponding group shows an entry in contrast to groups without targets. To reconstruct the scene compressive sensing techniques offers tools which takes these groups (also called block-sparsity) into account [11], [33].

There exist several promising CS algorithms to deal with these circumstances, for instance group lasso [34], block orthogonal matching pursuit (BOMP) [33], the group-sparse BPDN, or even the SPGL1 offer the possibility to form groups [11]. If each entry of s is assigned to a group, where α_i denotes the index of group i, the group-sparse BPDN problem is then

$$\min \left(\sum_i \|s_{\alpha_i}\|_2 \right) \quad \text{subject to} \quad \|As - y\|_2 \ll \sigma \tag{29}$$

From each MN transmit/receive pair a measurement vector $y_n = [y_n(1), ..., y_n(L)]^T$, which consists of L samples, is transferred to the central processing stage. Here the measurement vector y of the entire sensor system is constructed by staggering all measurement into a single vector of size $MNL \times 1$:

$$y = \left[[y_1]^T, \ldots, [y_{MN}]^T \right]^T \tag{30}$$

The sensing matrix A_{NM} for each transmit-receive pair is constructed as usual. The overall sensing matrix which compromises all nodes is build by having these submatrices on the diagonal:

$$A = \begin{pmatrix} A_1 & & 0 \\ & A_j & \\ 0 & & A_{MN} \end{pmatrix} \tag{31}$$

This ensures that measurements from one sensor will not be influenced by the other sensors. This is also shown in Fig. 9 Before solving now the linear equation system given by Eqn. (29) groups have
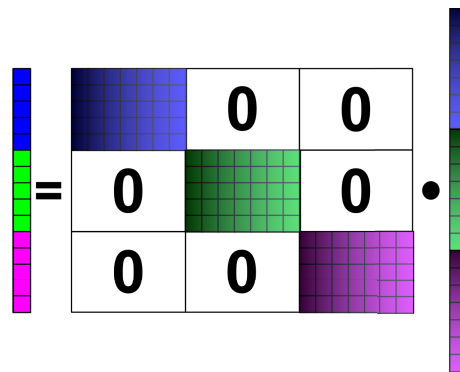

Fig. 9: Principle of setting up the sensing matrix A for a sensor network using group sparsity.

to be formed which combine identical target states from different sensors.

5.4.1 Numerical Results

To verify the proposed processing scheme several simulations have been carried out. Fig. 10 shows the setup consisting of a single transmitter, three receivers, and three targets, separated from each other by 30 m in x or y-direction and exhibit identical radar cross section. The transmitted pulse has a duration of 1.5 ms and was modulated in such a way to have a frequency bandwidth of 20 MHz, which corresponds to a monostatic range resolution of 7.5 m. Due to this all targets should be clearly resolved even by the matched filter approach. The center frequency for this simulation was $f_0 = 2.45$ GHz. As
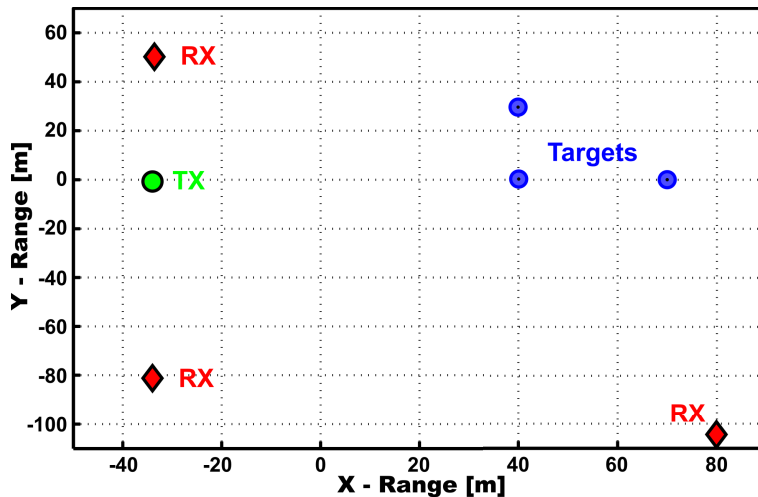
Fig. 10: Simulation setup consisting of 1 transmitter (TX), 3 receivers (RX), and 3 targets.

slowly moving target will produce a very low Doppler frequency shift ($v = 1$ m/s $\Rightarrow f_D = 16.67$ Hz) these objects will be modeled as a static target neglected for this simulation. However, by expanding the target state vector s and the CS sensing matrix A this effect can easily be incorporated.

For the matched filter approach the pulse compressed signal for each transmit-receive-pair is determined first. In the second step an additional filter was applied to reduce the sidelobes introduced by the modulation [35]. All pulse compressed signals have then to be transformed from the range-Doppler plane into the 2-dimensional x/y-plane of the distributed network. Fig. 11 shows the result of the incoherent integration over all received signals. It is clearly visible that the matched filter (MF) method is not able to detect all three targets. The pulse response of two targets overlap in such a way that they can't be resolved anymore. For the MF approach the beacon signal was digitized by 84400 samples and after pulse compression 2049 samples where considered for further evaluation.

For CS only the first $L = 800$ samples are used from each $N M$ transmit-receive pair to estimate the target state vector s. As Fig. 12 shows compressive sensing is able to estimate the position of all three targets with high accuracy.

Sensor fusion with the CS technique is even capable of dealing with much fewer samples. Fig. 13 shows the result obtained by CS with only $L = 50$ samples from each transmit-receive pair.
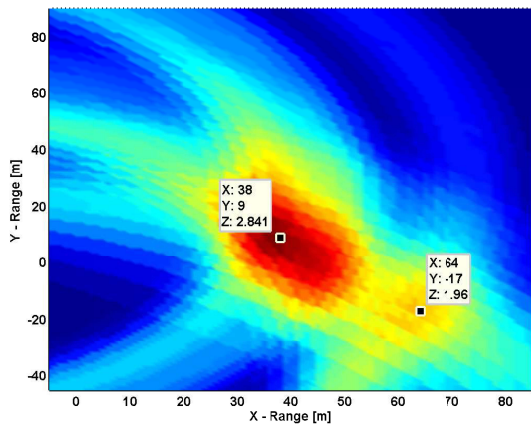
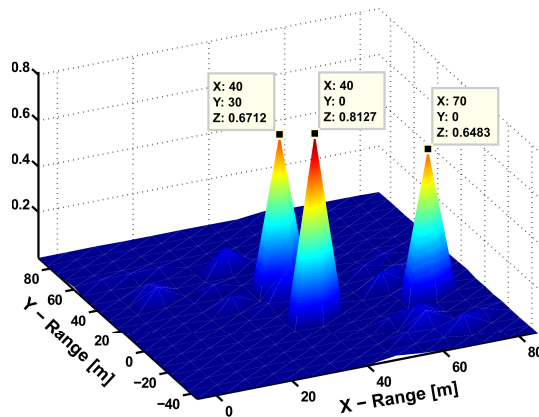Fig. 11: Target positions determined by the matched filter approach.

Fig. 12: Reconstructed state vector using SGPL1 and with $L = 800$ from each tx-rx-pair.
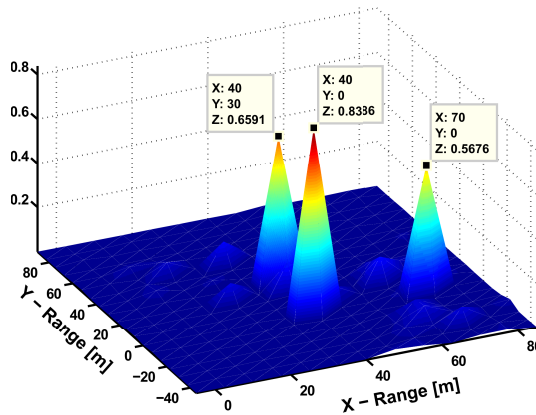

Fig. 13: Target state vector obtained by SGPL1 and $L = 50$ samples from each tx-rx-pair.

A second investigation with 4 transmitters and 4 receivers located on a semicircle and three targets now separated only by $15\,m$, as depicted in Fig. 14, demonstrates even more the advantage of CS in comparison to the matched filter approach.

As in the previous constellation, the matched filter approach is not able to detect all three targets, as shown in Fig. 15.

Compared to the incoherent matched filter the CS with its group sparsity is able to detect all three targets with high accuracy even with only $L = 50$ samples from each transmit-receive-pair, as shown in Fig. 16. This investigation shows the potential of CS for sensor networks. It allows a dramatic reduction of data transmission between sensor nodes and the central processing stage without performance degradation in target detection and localization accuracy.
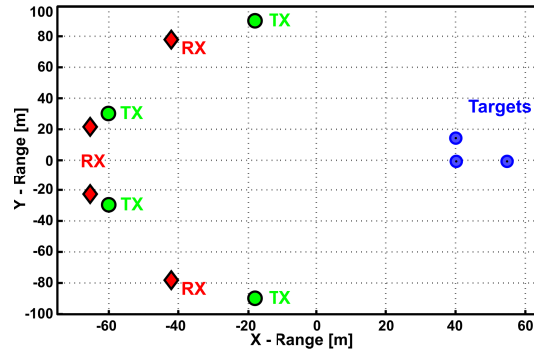
Fig. 14: Simulation setup consisting of 4 TX, 4 RX, and 3 targets.
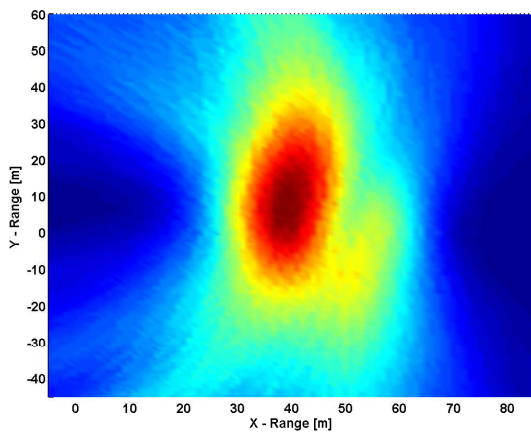

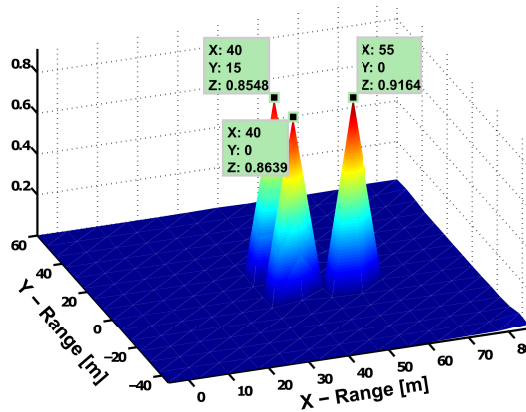Fig. 15: Target positions determined by the matched filter approach.

Fig. 16: Target state vector obtained by SGPL1 and $L = 50$ samples from each tx-rx-pair.

5.4.2   Noise

Besides efficient and faster solving it is of interest how the results of CS is influenced by noise, which is always present in real systems. For the following study nearly the same setup is used as depicted in Fig. 10 except that there is only one target present. For the simulation additive white Gaussian noise (AWGN) with different SNR levels are added to the *received* signal. MF and the CS SPGL1 algorithms are used to determine if there are targets and to estimate their locations. The performance of each algorithm is then characterised by determining the true-position-rate (TPR) for an average of 50 simulations for each SNR step. The TPR is the result from TruePositive over the sum of TruePositive and FalseNegative as described in [36]. Fig. 17 shows the result from this investigation.
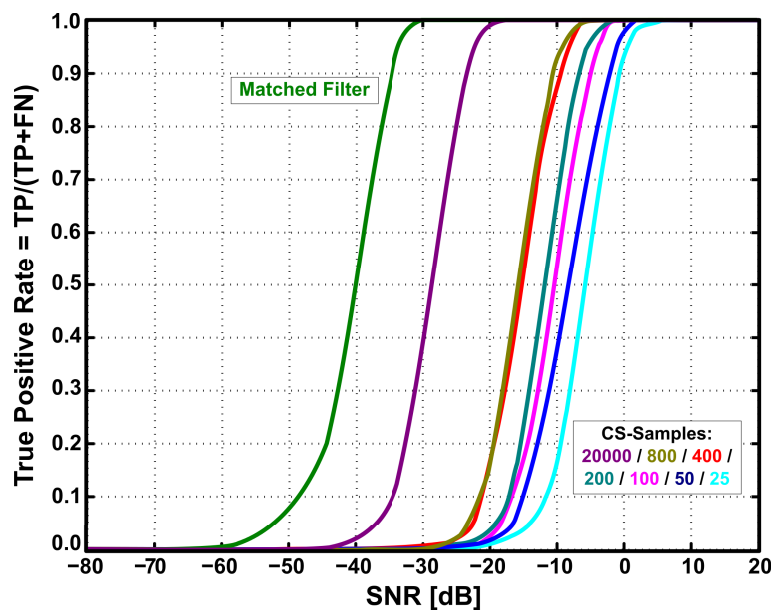

Fig. 17: True target position rate versus Signal-Noise-Ratio for different number of samples L.

As expected noise has a strong impact on the performance of both methods. The performance of the chosen CS algorithm (SPGL1) depends on two main parameters, first on the signal-noise-ratio and on L, the number of samples from each transmit-receive-pair. If no pulse compression is performed during the pre-processing of CS the target detection and localisation performance is up to 30 dB worse than the matched filter approach.

6.0   CONCLUSION

This paper has attempted to provide an introduction to compressive sensing techniques in the framework of radar and in the area of joined signal processing for a distributed sensor system. CS offers the

advantage of extracting information from less data than required by the Nyquist-Shannon criterium without decreasing the performance of the overall system. Investigations have shown that the detection performance depends on the knowledge of the involved transmit and receive nodes. If these can be modeled by a sensing matrix the number of samples which has to be transmitted from the receive nodes to a central processing stage can be dramatically reduced by compressive sensing techniques.

One should always keep in mind that compressive sensing is a discrete theory which assumes that all targets are located exactly on grid points of the discretization grid, which is determined by the range resolution of the system. In a distributed sensor network, built-up by mono- and bistatic radars, this is not always true.

7.0   REFERENCES

[1] *Hans Israelsson, Lars Ulander, Jan Askne, Johan Fransson, Per-Olov Frölind, Anders Gustavsson, and Hans Hellsten*, "Retrieval of forest stem volume using VHF SAR", IEEE Trans. GRS, Vol. 35, No. 1, pp.: 36–40, January 1997

[2] *Martina Daun*, "Deghosting in passive air surveillance systems", 11th International Radar Symposium (IRS), 16-18 June 2010, pp.: 1–8, Print ISBN: 978-1-4244-5613-0

[3] *David Donoho*, "Compressed Sensing", http://sys.cs.pdx.edu/trac/syn/export/36/CCS/ related%20work/CompressedSensing091604.pdf, September 14, 2004

[4] *David Wipf, Bhaskar Rao*, "l_0-norm Minimization for Basis Selection", Advances in Neural Information Processing Systems 17, 2005, pp.: 1513–1520

[5] *Emanuel Candès, Justin Romberg, Terence Tao*, "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information", IEEE Trans. Information Theory, Vol. 52, pp. 489–509, 2006

[6] *Richard Baraniuk*, "Compressive sensing", IEEE Signal Processing Magazine, Vol. 24, No. 4, pp.: 118–121, July 2007

[7] *Emanuel Candès, Michael Wakin*, "An Introduction to Compressive Sampling", IEEE Signal Processing Magazine, 2008, March, pp.: 21–30

[8] *Patrick Gill, Albert Wang, Alyosha Molnar*, "The In-Crowd Algorithm for Fast Basis Pursuit Denoising", IEEE Transaction on Signal Processing, Vol. 59, No. 10, pp.: 4594–4605, 2011, DOI: 10.1109/TSP.2011.2161292

[9] *Stéphane Mallat, Zhifeng Zhang*, "Matching pursuits with time-frequency dictionaries", IEEE Transaction on Signal Processing, Vol. 41, No. 12, pp.: 3397–3415, 1993, DOI: 10.1109/78.258082

[10] *Deanna Needell , Joel Tropp*, "CoSaMP: Iterative signal recovery from incomplete and inaccurate samples", Information Theory and Applications, 31 January 2008, San Diego, pp.: 1–25, arXiv:0803.2392

[11] *Ewout van den Berg and Michael P. Friedlander*, "Probing the Pareto frontier for basis pursuit solutions", SIAM Journal on Scientific Computing, Vol. 32, No. 2, 2008, pp.: 890–912, doi = 10.1137/080714488

[12] *Richard Baraniuk, Philippe Steeghs*, "Compressive Radar Imaging", IEEE Radar Conference, Waltham, MA, April 2007, pp.: 128—133

[13] *David Donoho, Michael Elad*, "Optimally sparse representation in general (nonorthogonal) dictionaries via ℓ_1 minimization", Proc. Natl. Acad. Sci., 100(5), pp.: 2197-2202, 2003

[14] *Albert Cohen, Wolfgang Dahmen, Ronald DeVore*, "Compressed sensing and best k-term approximation", J. Amer. Math. Soc., 22, pp.: 211–231, 2009

[15] *Joel A. Tropp and Anna C. Gilbert*, "Signal recovery from partial information via orthogonal matching pursuit", IEEE Trans. Inform. Theory, Vol. 53, No. 12, pp.: 4655-4666, 2007

[16] *Yonina Eldar, Gitta Kutyniok*, "Compressed Sensing: Theory and Applications", Cambridge University Press, 2012

[17] *Emanuel Candès, Terence Tao*, "Near optimal signal recovery from random projections: universal encoding strategies?", IEEE Trans. Inform. Theory, Vol. 52, 2006, pp.: 5406-–5425

[18] *Emanuel Candès, Justin Romberg, Terence Tao*, "Stable signal recovery from incomplete and inaccurate measurements", Comm. on Pure and Applied Math., Vol. 59, No. 8, pp.: 1207–1223, 2006

[19] *Jarvis Haupt, Robert D. Nowak*, "Signal reconstruction from noisy random projections", IEEE Trans. Information Theory, Vol. 52, No. 9, pp.: 4036–4048, 2006

[20] *David Donoho, Michael Elad, Vladimir Temlyakov*, "Stable recovery of sparse overcomplete representations in the presence of noise", IEEE Trans. Information Theory, Vol. 52, No. 1, pp.: 6–18, 2006.

[21] *Richard Baraniuk, Mark Davenport, Ronald DeVore, Michael Wakin*, "A simple proof of the restricted isometry property for random matrices", Constructive Approximation, December 2008, Vol. 28, No. 3, pp.: 253-263, DOI: 10.1007/s00365-007-9003-x

[22] *Holger Rauhut*, "Compressive Sensing and Structured Random Matrices", Theoretical Foundations and Numerical Methods for Sparse Recovery of Radon Series Comp. Appl. Math., de-Gruyter, 2010, pp.: 1–92

[23] *Robert Alexander Rankin*, "The closest packing of spherical caps in n dimensions", Proc. Glasgow Math. Assoc., vol. 2, pp: 139–144, 1955

[24] *L. R. Welch*, "Lower bounds on the maximum cross-correlation of signals", IEEE Trans. Inf. Theory, vol. 20, no. 3, pp: 397–399, 1974, DOI:10.1109/TIT.1974.1055219

[25] *Lam H. Nguyen, Thong T. Do, Trac D. Tran*, Sparse Model and Sparse Recovery with Ultra-wideband SAR Applications, 1st International Workshop on Compressed Sensing Applied to Radar (CoSeRa 2012), http://workshops.fhr.fraunhofer.de/cosera

[26] *Ali Cafer Gürbüz, James H. McClellan, Volkan Cevher*, "A compressive beamforming method", ICASSP 2008, IEEE International Conference on Acoustics, Speech and Signal Processing, 2008, pp.: 2617–2620, DOI: 10.1109/ICASSP.2008.4518185

[27] , *Joachim Ender*, "On Compressive Sensing Applied to Radar", Signal Processing, Vol. 90, No. 5, 2010, pp.: 1402–1414, DOI:10.1016/j.sigpro.2009.11.009

[28] *Aris Gretsistas, Mark D. Plumbley*, "A Multichannel Spatial Compressed Sensing Approach for Direction of Arrival Estimation", Latent Variable Analysis and Signal Separation, Lecture Notes in Computer Science, Vol. 6365, 2010, pp.: 458–465

[29] *Chris Baker*, "An Introduction to Multistatic radar", NATO SET-136 Lecture Series "Multistatic Surveillance and Reconnaissance: Sensor, Signals and Data Fusion", April 2009

[30] "*Matthias Weiß*, "Multi-Sensor Systems: Multiplicity Helps", RTO-EN-SET-157-2010 - Multi-sensor Fusion: Advanced Methodology and Applications, ISBN 978-92-837-0114-9

[31] *Daniel O'Hagan, Martin Ummenhofer, Heiner Kuschel, Jörg Heckenbach*, "A Passive/Active Dual Mode Radar Concept", 14th International Radar Symposium IRS 2013l, Dresden, pp.: 136–142

[32] , *Joachim Ender*, "A Compressive Sensing Approach to the Fusion of PCL and PET sensors", SET-187 Specialists Meeting on "Passive radar, challenges concerning theory and practice in military application", 13.-14. May 2013

[33] *Yonina Eldar, Patrick Kuppinger, Helmut Bolcskei*, "Block-Sparse Signals Uncertainty Relations and Efficient Recovery", IEEE Trans. on Signal Processing, Vol. 58, No. 6, June 2010, pp.: 3042–3054

[34] *X. Lv, G. Bi, C. Wan*, "The group lasso for stable recovery of block-sparse signal representations", IEEE Trans. on Signal Processing, Vol. 59, No. 4, pp.: 1371–1382

[35] , *Matthias Weiß*, "Passive WLAN radar network using compressed sensing ", IET International Conference on Radar Systems (Radar 2012), Glasgow, UK, 22-25 Oct. 2012, pp.: 1–6, DOI: 10.1049/cp.2012.1569

[36] *Tom Fawcett*, "ROC graphs: Notes and practical considerations for researchers", Machine Learning 31 (2004): pp.: 1–38